FUTURE MISSION STUDIES PRELIMINARY COMPARISONS OF SOLAR FLUX MODELS

ORIGINAL CONTAINS
COLOR SELUSTRATIONS

December 1991

FLIGHT DYNAMICS DIVISION

CODE 550

Developed Under the Direction of

Through

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December 1991

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ABSTRACT

The purpose of this document is to present the results of comparisons of the solar flux models. (The wavelength λ = 10.7 cm radio flux is the best indicator of the strength of the ionizing radiations such as solar ultraviolet and x-ray emissions that directly affect the atmospheric density thereby changing the orbit lifetime of satellites. accurate forecasting of solar flux F_{10.7} is crucial for orbit determination of spacecrafts.) The measured solar flux recorded by National Oceanic and Atmospheric Administration (NOAA) is compared against the forecasts made by Schatten, Marshall Space Flight Center (MSFC), and NOAA itself. This document also discusses the possibility of a combined linear, unbiased minimum-variance estimation that properly combines all three models into one that minimizes the variance. All the physics inherent in each model are combined. This is considered to be the dead-end statistical approach to solar flux forecasting before any nonlinear chaotic approach.

The research for this document was completed in December 1990.

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SECTION 1 - INTRODUCTION

This document is the first part of a sequence of preliminary studies of solar flux observed at the wavelength $\lambda=10.7$ cm range. The sequence starts with comparisons of different solar flux models and gradually leads to a critical stochastic approach, which further produces a geometric approach to the prediction of chaotic solar flux time series.

The analysis in this first sequence is based on the available forecasts by Schatten (at Goddard Space Flight Center (GSFC)) (Reference 1), Marshall Space Flight Center (MSFC) (Reference 2), and National Oceanic and Atmospheric Administration (NOAA) (Reference 3). The comparisons are made against actual observed values that are collected by NOAA (Reference 3).

The observable radio spectrum extends from 1 centimeter (cm) to 10 meters (m). Like the optical spectrum, the radio spectrum is limited on its short wavelength end by absorption in the Earth's atmosphere (by molecules of oxygen and water vapor). On the long wavelength end, the lower atmosphere is always transparent, even on cloudy days. But a high layer, called the ionosphere, begins to interfere at around $\lambda = 10$ m (References 4 and 5).

The radio waves are radiated by fast-moving electrons in the highly ionized gases of the outer solar atmosphere. Ionized gases, which are fully transparent to visible light, however, may be opaque to radio waves at certain wavelengths. The opacity depends on the density of ionized gas. In the solar chromosphere, where density is high, the gases are completely opaque to meter wavelengths; only the centimeter waves can escape the Sun to reach the Earth. The Sun that is observed is only the visible Sun; it appears larger in the radio region (that is, the appearance dimension is proportional to the wavelength).

There is a strong correlation between sunspots and the solar flux $F_{10.7}$ because probably most of the enhanced radiation comes from limited areas of the Sun where there are active sunspots. The activity depends on wavelength of radiated solar flux. For waves shorter than 3 cm, the intensity is steady. From 3 to 60 cm, often called decimeter range, the intensity shows occasional short-lived increases. tend to last for a few minutes. The decimeter intensity also shows a slowly varying component that tends to exhibit a 27-day period associated with solar rotation (Reference 6) and rises from the vicinity of active sunspot regions. Large sporadic outbursts, lasting for minutes, occur often in association with the bigger solar flares. A millionfold increase in intensity within a few seconds has been observed. (This will be studied as a part of the sequence of the solar flux analysis by identifying the abrupt changes as one of seven Thom's "elementary catastrophes.") (See Section 5 for recommendations.)

It is necessary to study solar flux and accurately forecast it to perform accurate orbit determination for a spacecraft. The orbit lifetime is a function of atmospheric drag force; this force is a function of atmospheric density, which itself is a function of solar flux.

Section 2 is devoted mostly to graphical analysis of solar flux data. Section 3 describes the statistical techniques to compare different forecasting models by confidence interval methods. Section 4 introduces a linear, unbiased minimum-variance estimation and combines three important models into one. Section 5 is devoted to conclusions and recommendations for future investigations.

SECTION 2 - SOLAR FLUX F_{10.7} PREDICTION

2.1 MATHEMATICAL PERSPECTIVES

For satellite orbit lifetime prediction, one has to evaluate the drag force that continually results in satellite orbit decay. By applying the fundamental principles of fluid mechanics, the drag force is written in the following form:

$$|\overrightarrow{F}| = \frac{1}{2} \rho v^2 C_d A \qquad (2-1)$$

where ρ is the atmospheric density, which is a complicated function of solar flux in different density models of the atmosphere. The velocity of the spacecraft is indicated by $\overrightarrow{\nu}$, and the other variables are properties of the spacecraft. These properties are drag coefficient C_d and scattering cross section A.

It is very clear from the above equation that, given the drag coefficient $C_{\widehat{d}}$ and the scattering cross section A, one can easily calculate the drag force $|\widehat{F}|$ if the density of the atmosphere is known. Since the atmospheric density is sensitive to solar activity, most of the density models are complicated functions of solar flux. The motivation for studying solar flux prediction models (other than solar astronomy) is the accurate satellite orbit lifetime prediction.

2.2 GRAPHICAL ANALYSIS OF DATA

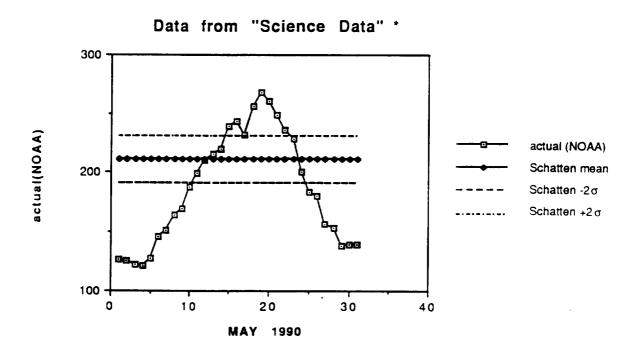
The first part of this study compares Schatten solar flux forecasts with National Oceanic and Atmospheric Administration (NOAA) actual solar flux values. Schatten predictions are modified inconsistently, but at least once every 3 months. NOAA also forecasts short-term predictions that are modified consistently every week. Schatten's

latest values were distributed on September 1, 1990, which includes long-term predictions starting from September 1990 to September 2008. His next latest predictions were distributed on May 25, 1990, which includes predictions starting May 1990 to August 1990 and on to April 2012. Therefore his May 25 version, which includes May, June, July, and August 1990 predictions, is by far the best he could do. Thus this analysis was done on his best predictions (May, June, July, and August 1990).

As seen from the graphs in Figures 2-1 to 2-10, the 30-day mean of the actual (NOAA) values are always less than +2 sigma value of Schatten and in most cases are even smaller than Schatten (mean). They are closer to -2 sigma value or even smaller than that. This may mean that the mission analysis is being too conservative by using +2 sigma value consistently.

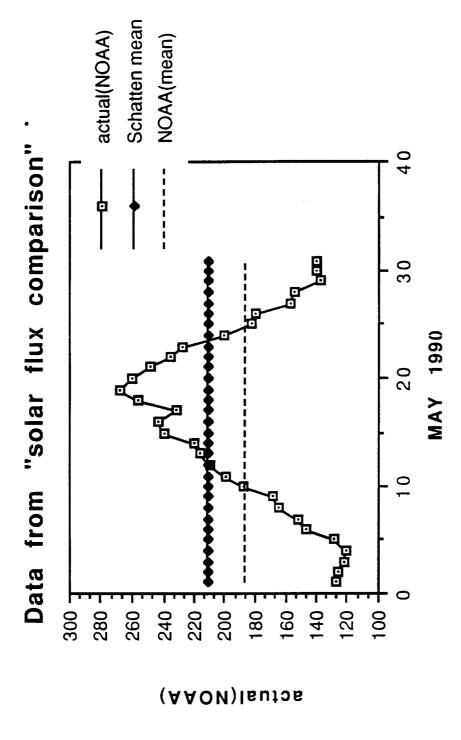
2.3 DESCRIPTION OF GRAPHS

Figures 2-1 to 2-10 present the actual solar flux values and different forecasts for the months of May, June, July, and August 1990. The forecasts are the best updates for those months. Figures 2-11 to 2-14 present the actual solar flux values and their different averages and different forecasts for a period of 2 years (October 15, 1988 to September 17, 1990). The actual data are daily values, and the long-term prediction models (Schatten and MSFC) are monthly values. Figures 2-15 and 2-16 are the confidence intervals for Schatten and MSFC forecasts, the nominal and the +2\sigma, respectively. Figures 2-17 to 2-19 are the actual solar flux values and their averages for three different timespans. The statistical analysis performed to get the confidence intervals is discussed in Section 3. It should be noted that all the units for the solar flux values are in units of



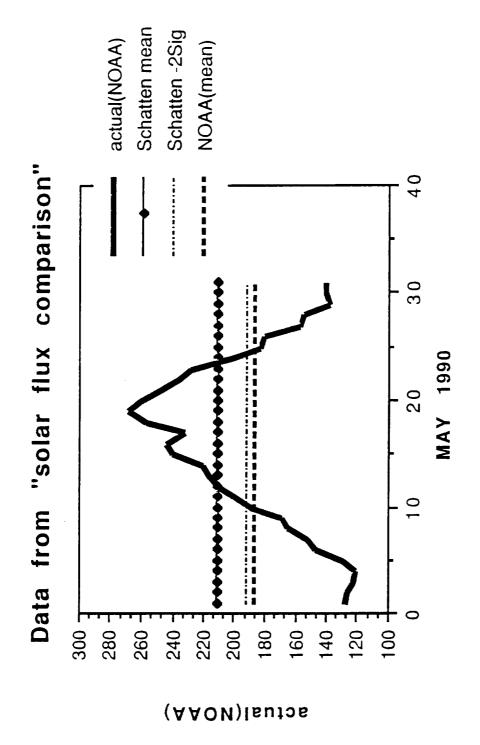
* Science data is a data file with its data printed by running a cricket graph on the Macintosh.

Figure 2-1. Plot of Solar Flux Values and Schatten Nominal, +2 Sigma, and -2 Sigma Predictions (May 1990)



Plot of Solar Flux Values, Schatten Nominal Prediction, and NOAA 27-Day Nominal Predictions (May 1990) Solar flux comparison is a data file with its data printed by running a cricket graph on the Macintosh. Figure 2-2.

2-4



Predictions, and NOAA 27-Day Nominal Predictions (May 1990) Plot of Solar Flux Values, Schatten Nominal and -2 Sigma Figure 2-3.

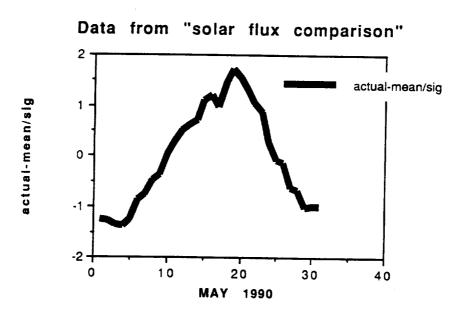
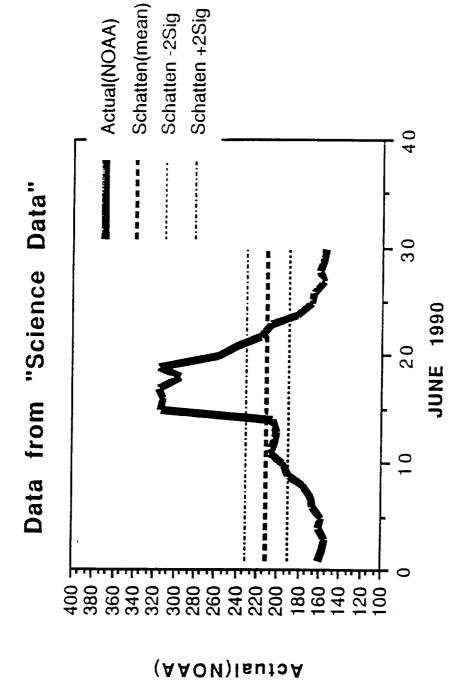
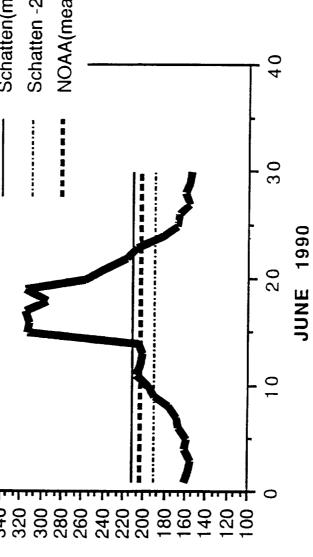


Figure 2-4. Plot of Solar Flux Values Minus the Mean Value Normalized by Standard Deviation (May 1990)

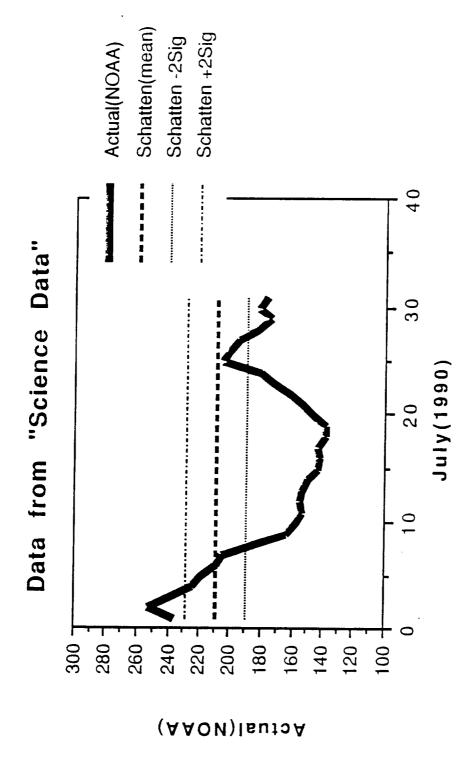


Plot of Solar Flux Values and the Schatten Nominal, +2 Sigma, and -2 Sigma Predictions (June 1990) Figure 2-5.

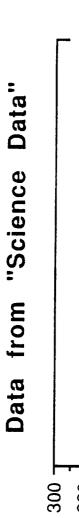


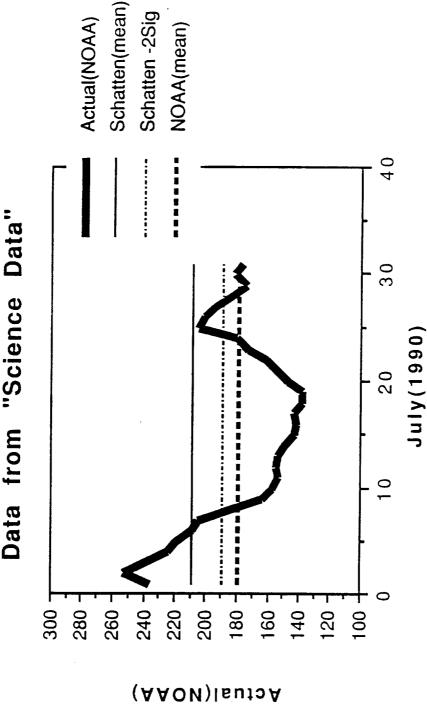


Plot of Solar Flux Values and the Schatten Nominal, -2 Sigma Predictions, and the NOAA 27-Day Nominal Predictions (June 1990) Figure 2-6.



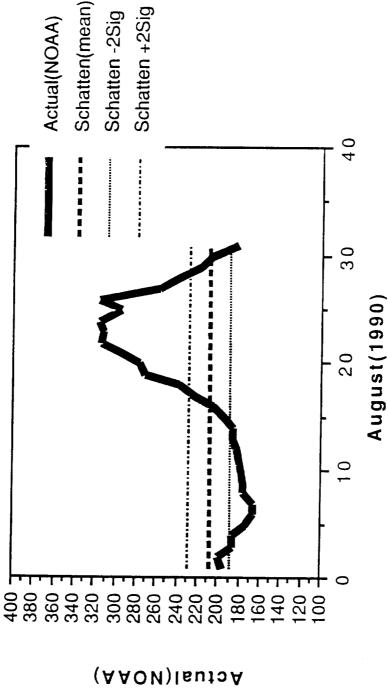
Plot of Solar Flux Values and the Schatten Nominal, +2 Sigma, and -2 Sigma Predictions (July 1990) Figure 2-7.



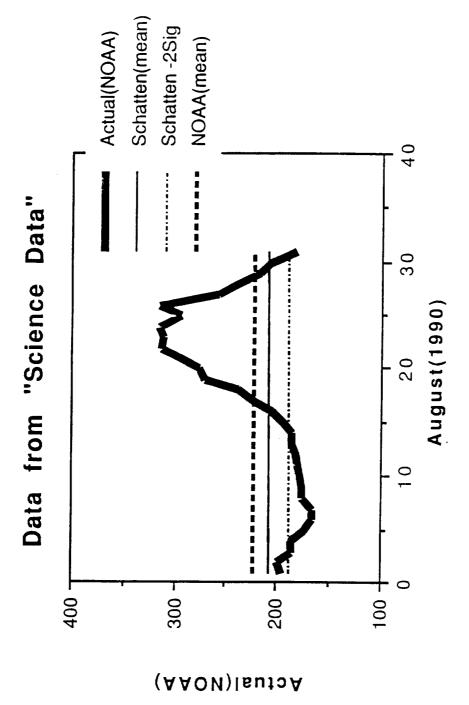


Plot of Solar Flux Values, the Schatten Nominal, -2 Sigma Predictions, and the NOAA 27-Day Nominal Predictions (July 1990) Figure 2-8.

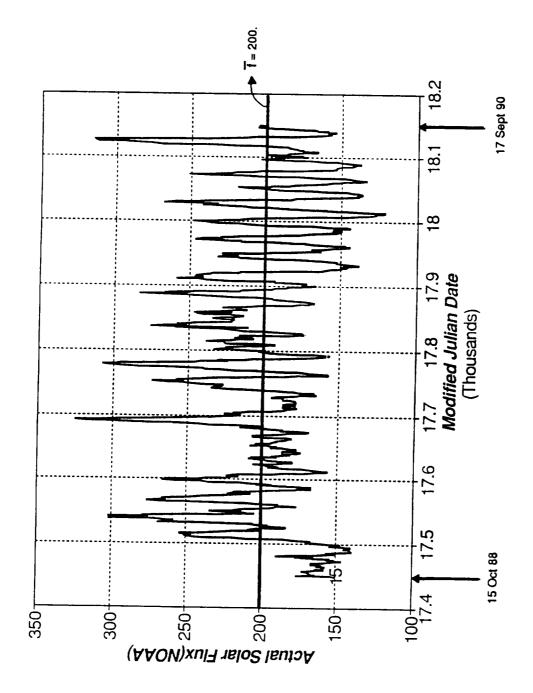




Plot of Solar Flux Values and the Schatten Nominal, -2 Sigma, and +2 Sigma Predictions (August 1990) Figure 2-9.



Plot of Solar Flux Values, the Schatten Nominal, -2 Sigma, and NOAA 27-Day Nominal Predictions (August 1990) Figure 2-10.



Plot of Actual NOAA Solar Flux Values for October 15, 1988, to September 17, 1990 Figure 2-11.

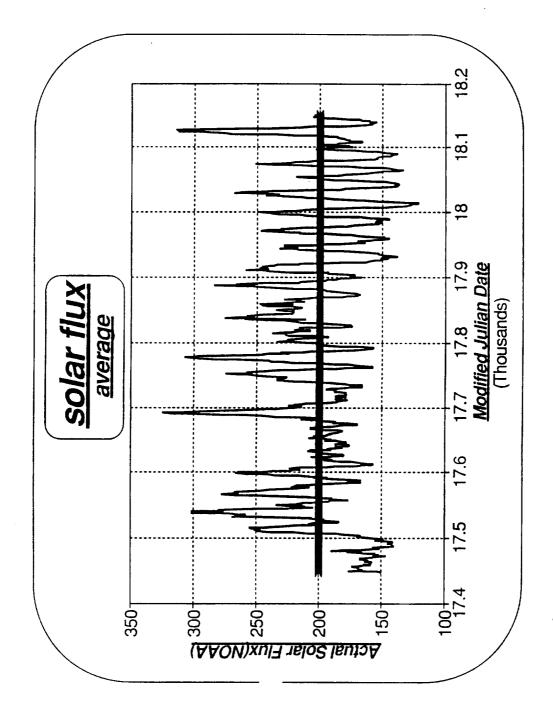
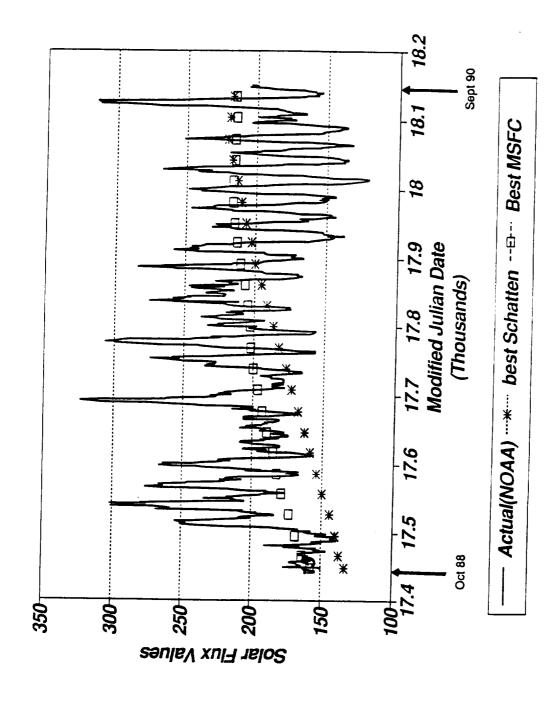


Figure 2-12. Plot of Actual Solar Flux Values and Mean Value



Plot of Solar Flux Values and the Nominal Best Schatten and MSFC Predictions Figure 2-13.

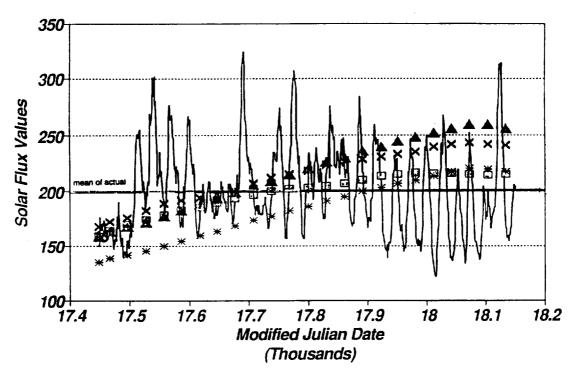
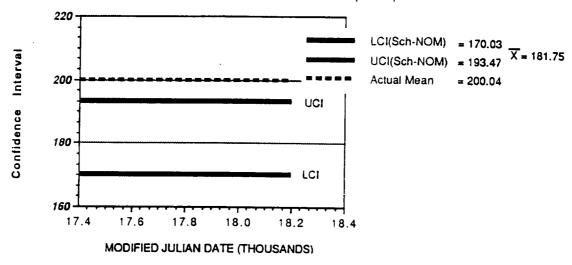


Figure 2-14. Plot of Solar Flux Values, the Schatten Nominal and +2 Sigma Predictions, and the MSFC Nominal and 97.7 Percent Predictions

Schatten Nominal Confidence Interval(95%)



MSFC Nominal Confidence Interval (95%)

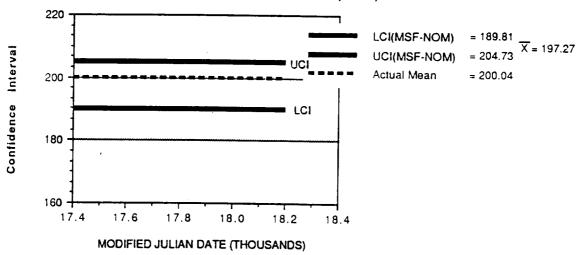
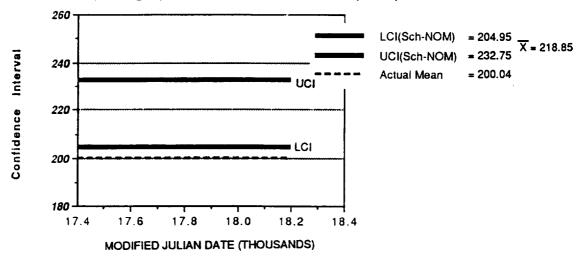


Figure 2-15. Plot of the Nominal Schatten and MSFC 95-Percent Predictions Confidence Interval and the Average of the Actual Solar Flux Values

Schatten(+2Sigm) Confidence Interval(95%)



MSFC 97.7 Confidence Interval (95%)

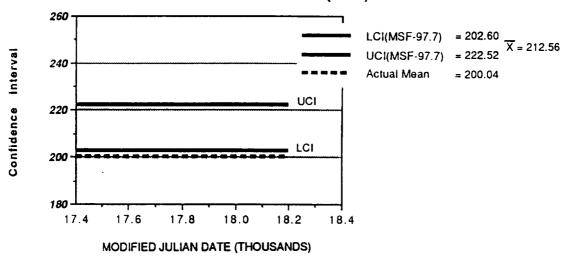
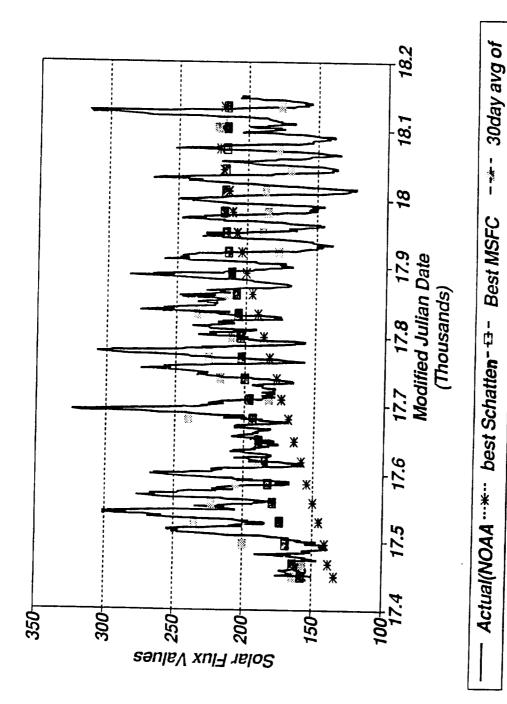
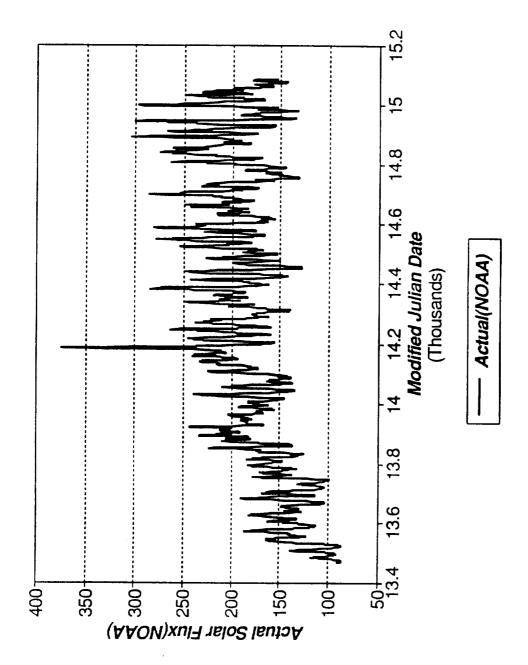


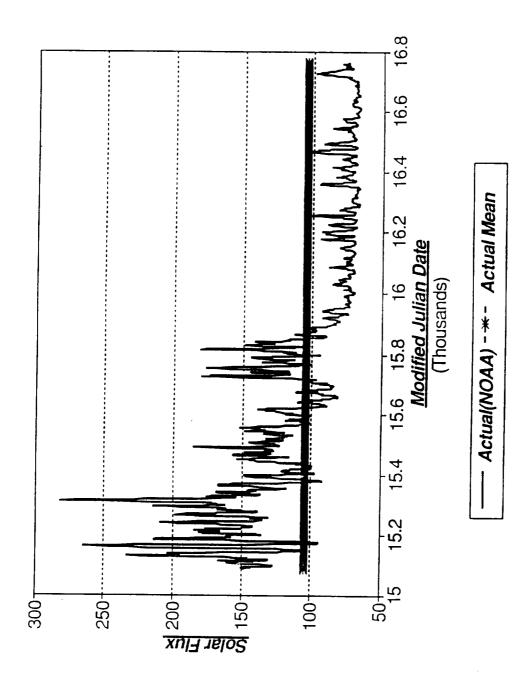
Figure 2-16. Plot of the Schatten +2 Sigma and MSFC 97.7-Percent Predictions Confidence Interval and the Average of the Actual Solar Flux Values



to September 17, 1990, and the Best Nominal Schatten and MSFC Prediction With the 30-Day Partitioned Average Plot of Actual Solar Flux Values From October 15, 1988, of Actual Solar Flux Values Figure 2-17.



Plot of the Actual Solar Flux Values for a Different Timespan (MJD 13400 to 15200) Figure 2-18.



Plot of the Actual Solar Flux Values and Its Average for a Different Timespan (MJD 15000 to 16800) Figure 2-19.

watt/m²/Hz x 10^{-22} for the range λ = 10.7 cm wavelength, and the horizontal axis is time in modified Julian date.

In order to compare forecast models of solar flux, one can compare the forecasts of solar flux (F_{10.7}) made for the timespan that the actual solar flux values are available. Every forecast will result in an interval (with a certain percentage of confidence). The actual population mean of data will fall within that confidence interval. This interval can be calculated for each forecast model. To compare forecast models, the question is whether the confidence interval encloses the population mean of the actual solar flux values or not. If it does, 95.5 percent of the time the predicted value is within the confidence interval; thus, it is a good forecast. The mathematical analysis of this procedure is presented in the next sections.

SECTION 3 - STATISTICAL ANALYSIS

3.1 CHECK OF A HYPOTHESIS

Sample values are often used as estimators for parameters of random variables. However, these procedures result only in point estimates for a parameter of interest; no indication is provided about how closely a sample value estimates the parameter. A more meaningful procedure for estimating parameters of random variables involves the estimation of an interval, as opposed to a single point value, which will include the parameter being estimated with a known degree of uncertainty. For example, consider the case where the sample mean \bar{x} computed from N independent observations of a random variable x is being used as an estimator for the mean value $\mu_{\text{\tiny \mbox{\tiny w}}}.$ It is usually more desirable to estimate μ in terms of some interval d, such as $\overline{x} \pm d$, where there is a specified uncertainty that $\mu_{\mathbf{x}}$ falls within that interval. Such intervals can be established if the sampling distribution of the estimator in question is known (Reference 7).

It can be shown that probability statements can be made concerning the value of a sample mean $\overline{\mathbf{x}}$ as follows.

prob
$$\left[z_{1-(\alpha/2)} < \frac{(\bar{x} - \mu_{\bar{x}}) \sqrt{N}}{\sigma_{\bar{x}}} \le z_{\alpha/2}\right] = 1 - \alpha$$
 (3-1)

$$\operatorname{prob}\left[\frac{1}{x} > \left(\frac{\sigma_{x} z_{\alpha}}{\sqrt{N}} + \mu_{x}\right)\right] = \alpha \qquad (3-2)$$

where \overline{x} = sample mean, μ_{x} = population mean, N = number of observations, σ_{x} = sample standard deviation, d = uncertainty length, α = probability measure, and z_{α} = desired percentage of confidence.

As the sample size N becomes large, the sampling distribution of the sample mean \bar{x} approaches a normal distribution regardless of the distribution of the original variable x. For a sample, the probability statement would be either 1 or 0, i.e.,

$$\operatorname{prob}\left[z_{1-(\alpha/2)} < \frac{(\overline{x} - \mu_{x}) \sqrt{N}}{\sigma_{x}} \le z_{\alpha/2}\right] = \begin{cases} 0 \\ 1 \end{cases} \tag{3-3}$$

As the value α becomes small (as the interval between $z_{1-(\alpha/2)}$ and $z_{\alpha/2}$ becomes wide), the probability is more likely to be unity rather than zero. In slightly different terms, if many different samples were repeatedly collected and a value \bar{x} were computed for each sample, one would expect

$$prob [] = \begin{cases} 0 \\ 1 \end{cases}$$

to fall within the noted interval for about $1-\alpha$ of the samples. In this context a statement can be made about an interval within which one would expect to find the quantity

$$\frac{(\overline{x} - \mu_{X}) \sqrt{N}}{\sigma_{X}}$$
 (3-5)

with a small degree of uncertainty. Such a statement is called a confidence statement. The interval associated with the confidence statement is called a confidence interval.

For the case of the mean value estimate, a confidence interval can be established for the mean value $\mu_{\mathbf{x}}$ based upon the sample value $\overline{\mathbf{x}}$ by rearranging terms in the previous equation as follows:

$$\left[\overline{x} - \frac{\sigma_{x} z_{\alpha/2}}{\sqrt{N}} \le \mu_{x} < \overline{x} + \frac{\sigma_{x} z_{\alpha/2}}{\sqrt{N}} \right]$$
 (3-6)

3.2 CHI-SQUARE DISTRIBUTION

For variance $\sigma_{\mathbf{x}}^2$ based on the sample variance s^2 for a sample size of N = 31, one would use chi-square distribution χ^2

$$\left[\frac{n s^{2}}{x_{n;\alpha/2}^{2}} < \sigma_{x}^{2} < \frac{n s^{2}}{x_{n;1-\alpha/2}^{2}}\right] \qquad n = N - 1$$
 (3-7)

$$\left[\frac{30s^{2}}{x_{30;\alpha/2}^{2}} < \sigma_{x}^{2} < \frac{30s^{2}}{x_{n;1-\alpha/2}^{2}}\right] \qquad n = 31 - 1 = 30$$
(3-8)

From a standard statistical table called, "Percentage Points of Chi-Square Distribution" (Reference 8) the value of $x_{n;\alpha/2}^2$ given α can be found.

For the value of α = 0.10, 1 - $\alpha/2$ = 0.95, and $\alpha/2$ = 0.05, $\chi^2_{30;\alpha/2}$ = 43.77 and $\chi^2_{30;1-\alpha/2}$ = 18.49.

So the interval reduces to

$$[0.6854s^{2} \le \sigma_{x}^{2} < 1.622s^{2}]$$
 (3-9)

Calculate the sample mean $\overline{\mathbf{x}}$ and the sample variance \mathbf{s}^2 to find both intervals.

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		\sim
		$\overline{}$

SECTION 4 - LINEAR, UNBIASED MINIMUM-VARIANCE ESTIMATION (LUMVE)

Schatten predictions are now adopted by GSFC because they apparently did a good job at some periods of time. But the conclusion from the data for the past 2 years is that MSFC predictions were closer to the actual solar flux values. This conclusion shows that it is not possible to compare the accuracy of two forecasting models (which use stochastic methods) when they try to model a time series that is inherently chaotic (existence of a structure in data). Therefore the best method is to combine all the models into one. This method is investigated in this section under linear, unbiased minimum-variance estimation (LUMVE). In this method, the three solar forecasting models—NOAA, MSFC, and Schatten predictions—are combined into one that minimizes the variance.

4.1 MATHEMATICAL FORMULATION

The LUMVE ensures that the variance of the combined solar flux predictions is the smallest that can be achieved for any linear, unbiased combination of the individual predictions. This method was used because it was demonstrated that MSFC predictions were closer to the actual solar flux than Schatten's for the past 2 years.

Solar flux is an inherently unpredictable phenomenon, and stochastic methods used by Schatten, MSFC, and NOAA cannot produce good predictions. Therefore, the dead-end approach before an analytic (not stochastic) nonlinear, chaotic approach is the linear, unbiased minimum-variance approach (References 9 and 10).

Let

 σ_n = standard deviation of NOAA prediction

 σ_{m} = standard deviation of MSFC prediction

 $\sigma_{_{\mathbf{S}}}$ = standard deviation of Schatten prediction

Consider a linear combination of the three flux predictions. Using normalized coefficients

$$\hat{f} = \frac{b_n f_n + b_m f_m + b_s f_s}{b_n + b_m + b_s}$$
 (4-1)

where b_n , b_m , and b_s are coefficients for NOAA, MSFC, and Schatten forecast of solar flux f_n , f_m , and f_s , respectively.

Define

$$a_n = \frac{b_n}{b_n + b_m + b_s}$$
 (4-2)

$$a_{m} = \frac{b_{m}}{b_{n} + b_{m} + b_{s}}$$
 (4-3)

$$a_{s} = \frac{b_{s}}{b_{n} + b_{m} + b_{s}}$$
 (4-4)

thus

$$\hat{f} = a_n f_n + a_m f_m + a_s f_s$$
 (4-5)

Now the problem is to select values of a_n , a_m , and a_s that will yield the best \hat{f} (the closest value to the mean of the actual data).

The variance of \hat{f} is

$$V(\hat{f}) = a_n^2 \sigma_n^2 + a_m^2 \sigma_m^2 + a_s^2 \sigma_s^2 + \underbrace{\text{covariance terms}}_{0 \text{ for independent predictions}}$$

$$(4-6)$$

By imposing $E(\widehat{f}) = f$ (where f is the desired actual mean), and forcing the parameters

$$E(\hat{f}) = E(f_n) = E(f_m) = E(f_s) = f$$
 (4-7)

which is a renormalization technique. Then,

$$E(\hat{f}) = a_n E(f_n) + a_m E(f_m) + a_s E(f_s)$$
 (4-8)

and

$$f = a_n f + a_m f + a_s f$$
 (4-9)

$$1 = a_n + a_m + a_s$$
 (4-10)

Equation (4-6) then becomes

$$V(\widehat{f}) = a_n^2 \sigma_n^2 + a_m^2 \sigma_m^2 + (1 - a_n - a_m)^2 \sigma_s^2$$
 (4-11)

or

$$V(\hat{f}) = a_n^2 \left(\sigma_n^2 + \sigma_s^2\right) + a_m^2 \left(\sigma_m^2 + \sigma_s^2\right) - 2a_n \sigma_s^2 - 2a_m \sigma_s^2 + 2a_n a_m \sigma_s^2 + \sigma_s^2$$

$$(4-12)$$

If the variance $V(\hat{f})$ is minimized (the ideal case), that is

$$\frac{\partial V(\hat{f})}{\partial a_n} = 0 (4-13)$$

then $a_n \left(\sigma_n^2 + \sigma_s^2 \right) + a_m \sigma_s^2 = \sigma_s^2$, and

$$\frac{\partial V(\hat{f})}{\partial a_m} = 0 (4-14)$$

then $a_n(\sigma_s)^2 + a_m(\sigma_m^2 + \sigma_s^2) = \sigma_s^2$; therefore

$$a_{n} = \frac{\sigma_{m}^{2} \sigma_{s}^{2}}{\sigma_{n}^{2} \sigma_{m}^{2} + \sigma_{n}^{2} \sigma_{s}^{2} + \sigma_{m}^{2} \sigma_{s}^{2}}$$
(4-15)

$$a_{m} = \frac{\sigma_{n}^{2} \sigma_{s}^{2}}{\sigma_{n}^{2} \sigma_{m}^{2} + \sigma_{n}^{2} \sigma_{s}^{2} + \sigma_{m}^{2} \sigma_{s}^{2}}$$
(4-16)

$$a_{s} = \frac{\sigma_{n}^{2} \sigma_{m}^{2}}{\sigma_{n}^{2} \sigma_{m}^{2} + \sigma_{n}^{2} \sigma_{s}^{2} + \sigma_{m}^{2} \sigma_{s}^{2}}$$
(4-17)

By dividing by σ_n^2 σ_m^2 σ_s^2

$$a_{n} = \frac{1/\sigma_{n}^{2}}{1/\sigma_{n}^{2} + 1/\sigma_{m}^{2} + 1/\sigma_{s}^{2}} = \frac{\omega_{n}}{\omega_{n} + \omega_{m} + \omega_{s}}$$
(4-18)

$$a_{m} = \frac{1/\sigma_{m}^{2}}{1/\sigma_{n}^{2} + 1/\sigma_{m}^{2} + 1/\sigma_{s}^{2}} = \frac{\omega_{m}}{\omega_{n} + \omega_{m} + \omega_{s}}$$
(4-19)

and

$$a_{s} = \frac{1/\sigma_{s}^{2}}{1/\sigma_{n}^{2} + 1/\sigma_{m}^{2} + 1/\sigma_{s}^{2}} = \frac{\omega_{s}}{\omega_{n} + \omega_{m} + \omega_{s}}$$
(4-20)

where

$$\omega_{n} = \frac{1}{\sigma_{n}^{2}}$$

$$\omega_{m} = \frac{1}{\sigma_{m}^{2}}$$

$$\omega_{s} = \frac{1}{\sigma_{s}^{2}}$$

$$f = \frac{\omega_{n}f_{n} + \omega_{m}f_{m} + \omega_{s}f_{s}}{\omega_{n} + \omega_{m} + \omega_{s}}$$

$$(4-21)$$

This approach requires only the ratios of the coefficients (a_n, a_m, a_s) and not the actual parameters. See Figure 4-1 for calculations of these parameters. The variances are calculated on the PC (IBM AT compatible) using the Quattro Pro program.

Can the same coefficients be used in the future or do these coefficients vary with time? If they vary in time are the variations predictable or not? Figures 4-1 through 4-10 show that these coefficients evolve in time in a predictable fashion. The reason they are predictable is that all the variations are already in the flux values and their adjust ment coefficients a_n , a_m , a_s do not vary violently and are predictable (Figures 4-1 through 4-10).

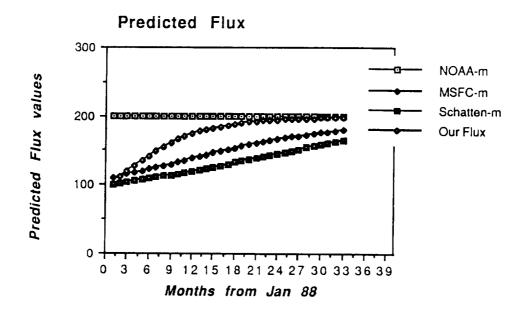
4.2 GRAPHICAL ANALYSIS OF DATA

The calculations of the required parameters in LUMVE presented in the previous section is performed here. The intermediate coefficients— a_n , a_m , and a_s —are calculated from Equations (4-15) through (4-21) and presented in Figure 4-1.

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	MOAA M	

	Intermediate	Intermediate calculations of equation (4-15) through equation (4-21)	เก (4-21)	
where	NOAA-m: MSFC-m: Schatten-m: NOAA-sn: MSFC-sn: Schatten: Sig-n "2: Sig-n "2: Sig-m "2: Sig-m "2: Sig-m "2:	NOAA nominal prediction f _m (Solar Flux) MSFC nominal prediction f _m (Solar Flux) Schatten nominal prediction f _s (Solar Flux) G n G m G s G n G m G m G m G m G m G m G m G m G m G m	DEN: An: As: As: An.fn: As: As: As: As: As: As: As: As:	a_1 a_2 a_3 a_4 a_4 a_5

Calculation of the Parameters Used in Linear, Unbiased Minimum-Variance Estimation (Running Procedure: Running Average of Data) Figure 4-1.



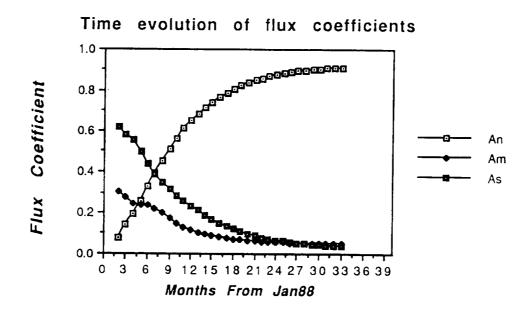
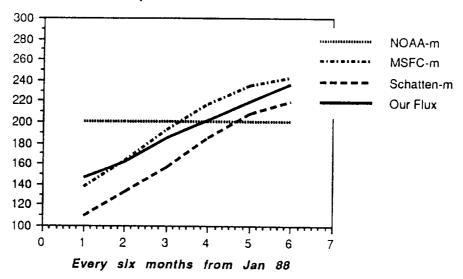


Figure 4-2. Predicted Flux by NOAA, MSFC, Schatten (Nominals) and the Result of Linear, Unbiased Minimum-Variance Estimation (Time Evolution of the Coefficients a_n , a_m , and a_s -Running Procedure)

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Calculation of the Parameters Used in Linear, Unbiased Minimum-Variance Estimation (6-Month Partition Procedure--Partitioned Average of Data) Figure 4-3.

Flux values predicted



Evolution of Flux coefficients

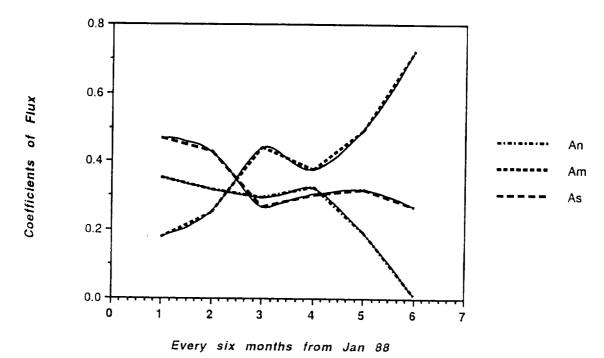
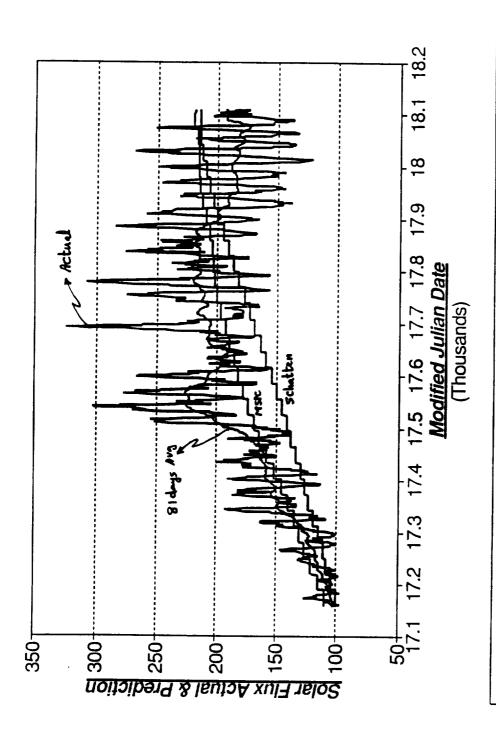


Figure 4-4. Plot of the Predicted Flux by NOAA, MSFC, Schatten (Nominal) and Result of Linear, Unbiased Minimum-Variance Estimation (Time Evolution of the Coefficients an, am, and as--Partitioned Procedure)



Plot of the Actual Solar Flux Values, Nominal Schatten and MSFC Predictions, and 81-Day Average Figure 4-5.

MSFC

Schatten

----. 81 days Avg ----

Actual

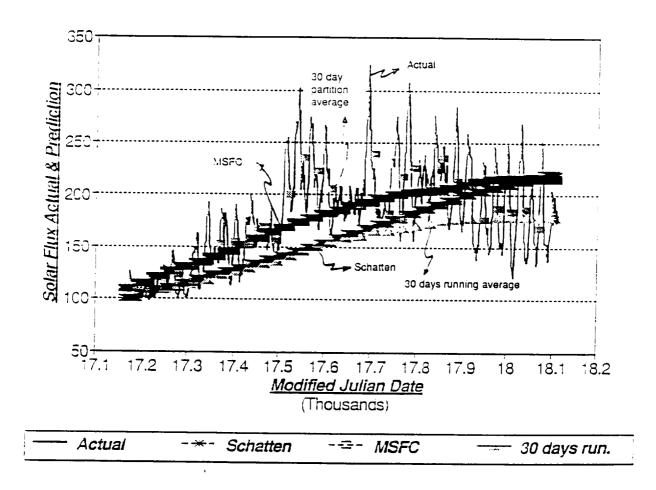


Figure 4-6. Plot of the Actual Solar Flux Values, Schatten, and MSFC Predictions With 30-Day Running Average and 30-Day Partitioned Average

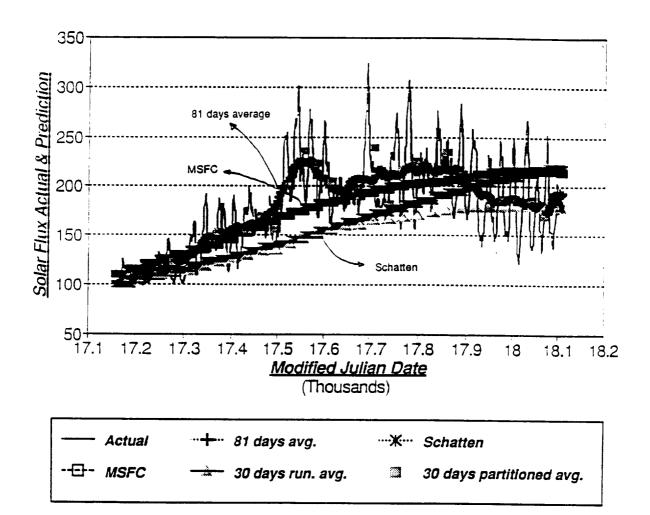
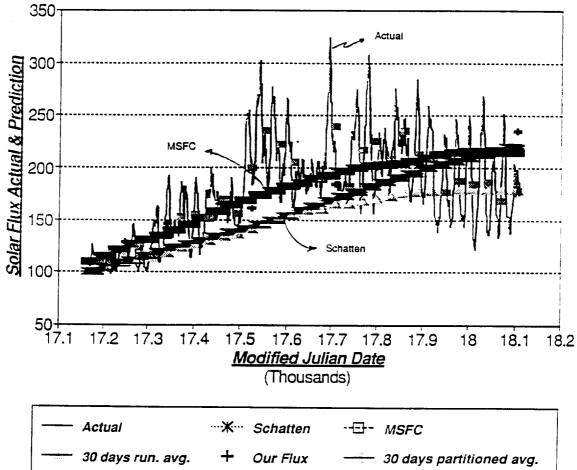


Figure 4-7. Plot of the Actual Solar Flux Values, Schatten, MSFC Predictions, and 81-Day Average With 30-Day Running Average and 30-Day Partitioned Average



Plot of the Actual Solar Flux Values, Figure 4-8. Schatten, and MSFC Predictions (30-Day Running Average; 30-Day Partitioned Average; and Linear, Unbiased Minimum-Variance Estimation by 6-Month Partitioned Procedure)

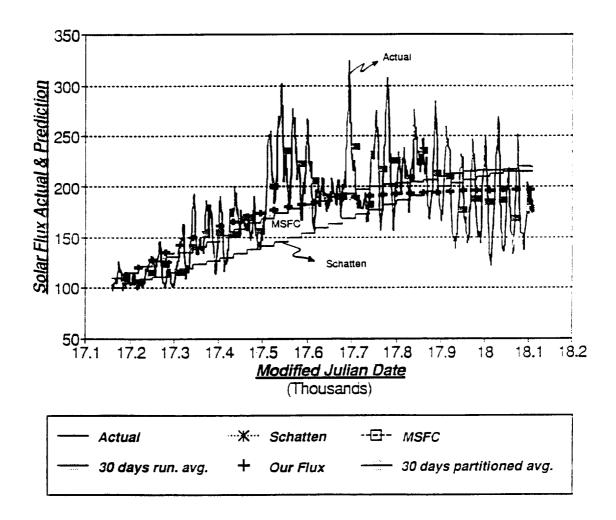
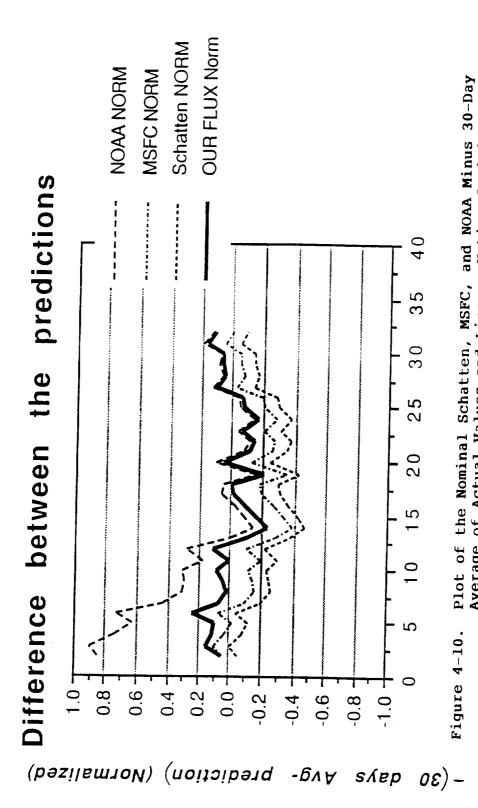


Figure 4-9. Plot of the Actual Solar Flux Values and the Schatten and MSFC Predictions With 30-Day Running Average and the Estimated Combined Flux With Partitioned Procedure



Average of Actual Values and Linear, Unbiased Minimum-

Variance Estimation (Normalized)

Finally the combined flux is calculated from Equation (4-21). Note that these equations require variances of NOAA, MSFC, and Schatten data that had been calculated using the Quattro Pro program.

The time evolutions of a_n , a_m , and a_s are presented in Figures 4-2 through 4-4, which indicate that their variations are not violent and are predictable in a sense. ures 4-5 through 4-8 present the actual solar flux values for a period of 2 years with MSFC and Schatten predictions and different kinds of averages, so that one can clearly see that MSFC predictions are closer to the actual values than Schatten predictions. Figure 4-9 presents the combined flux values, which are better than all the other models, because it apparently has all the inherent physics of the individual models built into it by the proper coefficients (a_n , a_m , and a_s). Figure 4-10 presents the normalized deviations of the predictions by individual models from the 30-day average of the actual solar flux values. It is clear from this graph that the combined solar flux prediction model varies within 20 percent of the actual values, whereas the other individual prediction models show variations much larger than 20 percent from the actual values.

SECTION 5 - CONCLUSIONS AND RECOMMENDATIONS

The statistical analysis using confidence intervals reveals that one cannot draw a general conclusion about which solar flux forecast model is better than the others among Schatten, NOAA, and MSFC. This is due to the fact that these models assume stochasticity (structurally random) in solar flux time series, which is chaotic (existence of an underlying structure in data).

Before employing the nonlinear, chaotic approach that will follow as the second sequence of the analysis (to be published in a different document), a combined LUMVE has been developed that properly combines all three models into one that minimizes the variance. All the physics inherent in each model are combined.

In the second part of these studies, solar flux as a chaotic time series will be studied and a particular route through which the dynamical time series becomes chaotic will be identified. The third part of the sequence is a critical stochastic approach to solar flux. In this part solar flux is studied through model identification, estimation, fitting, diagnostic checking, and mathematical forecasting for solar flux chaotic data.

The following future investigations are recommended:

• A Box-Jenkins type approach to solar flux time series model identification, estimation, fitting, diagnostic checking, and forecasting of solar flux. This is a method to classify the solar flux time series as one of the presently known models (moving average (MA) model, auto regressive (AR) model, and mixed auto regressive moving average (ARMA) model). Once the classification is made, proper forecastings seem possible.

- Chaotic approach to solar flux prediction. Since solar flux is shown to be chaotic, this approach would allow for the construction of an iterative manifold that can reconstruct the solar flux time series. To do this a sequence of studies should be performed such as
 - Finding Lyapunov spectrum from solar flux time series (Reference 11)
 - Forming attractors from solar flux time series and nonlinear signal processing using Neural Net
 - Extracting self similarity character and fractal structures from solar flux time series and modeling abrupt changes with Thom's elementary catastrophes (Reference 12)

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- 12. --, Towards Modeling the Formation of Abrupt Changes in Solar Flux by Thom's Catastrophe Theory (in preparation)

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